Black Hole Quasinormal Modes as Quantum Mechanical Bound States

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Perturbations of a Schwarzschild black hole reduce to a one-dimensional Schrödinger-type equation with an effective potential barrier. Unlike ordinary bound-state problems, quasinormal modes satisfy outgoing wave boundary conditions at infinity and ingoing wave boundary conditions at the black hole horizon, making the eigenvalue problem non-Hermitian and giving discrete complex frequencies. This paper explains the quantum-mechanical structure of these modes through the WKB approximation and the relation between potential barrier resonances and the bound states of inverted potentials.

INTRODUCTION

A quasinormal mode (QNM) is a characteristic oscillation of an open, dissipative system that decays with time. The associated frequency of a quasinormal mode is therefore complex, where the real part gives the physical oscillation frequency and the imaginary part gives the rate of damping. An example of such an "open" system is a black hole, from which radiation can escape to infinity and can also cross into its event horizon.

One important reason to study black hole QNMs is that they describe the stage after a black hole is disturbed. For example, after a binary black hole merger, the remnant black hole settles down by emitting gravitational waves in damped oscillations whose frequencies and decay times are determined by its properties like mass and spin. Quasinormal modes are the characteristic tones of the black hole [9, 10].

To describe the damping of QNMs mathematically, let $\Psi(t, x)$ denote the full time-dependent wave perturbation, and let $\psi(x)$ denote its spatial component. For a single mode, we separate the time and spatial dependence by writing

$$\Psi(t, x) = e^{i\omega t}\psi(x), \qquad \omega = \omega_R + i\Gamma. \tag{1}$$

Then we can write

$$e^{i\omega t} = e^{i\omega_R t}e^{-\Gamma t}.$$

Here, ω_R sets the physical oscillation frequency and $\Gamma > 0$ sets the damping rate.

Interestingly, the radial perturbation equations of a Schwarzschild (uncharged and non-rotating) black hole take the same mathematical form as a one-dimensional time-independent Schrödinger equation. The black hole does not produce a potential well, but instead produces an effective potential barrier. The QNM problem can therefore be analyzed in a way similar to a quantum resonance problem.

In this paper, I will first show how the Regge-Wheeler equation describing Schwarzschild perturbations becomes a one-dimensional barrier problem. Second, I will explain the quasinormal mode boundary conditions that make the problem non-Hermitian, giving complex eigenvalues. Third, I will show how the WKB approximation can be used near the top of the barrier potential to find the leading QNM spectrum in terms of the barrier height and curvature. Fourth, I will show how Ferrari and Mashhoon relate the QNMs of an analytically solvable barrier to bound states of an inverted potential, specifically for the inverted Pöschl-Teller and Eckart potentials. Finally, I will touch on recent work by Völkel that extends this bound state correspondence to spectra that are known through numerical computation, and shows that the original inverted Regge-Wheeler bound states give information about overtones and stability.

THE REGGE-WHEELER EQUATION AS A SCHRÖDINGER EQUATION

The Schwarzschild metric for a non-rotating, uncharged black hole of mass M is

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2, \tag{2}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ and $G = c = 1$. In 1957, Regge and Wheeler studied small nonspherical perturbations of this geometry and separated the angular dependence into spherical harmonics labeled by ℓ and m [1]. They separated the perturbations into odd-parity and even-parity sectors. They found that for the odd-parity sector, the radial equation can be written as one second-order equation after introducing the tortoise coordinate

$$r_* = r + 2M\ln\left(\frac{r}{2M} - 1\right), \qquad \frac{dr_*}{dr} = \left(1 - \frac{2M}{r}\right)^{-1}. \tag{3}$$

The black hole horizon $r = 2M$ is mapped to $r_* \to -\infty$, while spatial infinity is mapped to $r_* \to +\infty$.

The Regge-Wheeler equation is

$$\frac{d^2\psi}{dr_*^2} + \left[\omega^2 - V_{\mathrm{RW}}(r)\right]\psi = 0, \tag{4}$$

with the Regge-Wheeler potential:

$$V_{\rm RW}(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right], \qquad \ell \geq 2. \tag{5}$$

This has the same differential equation form as a one-dimensional stationary Schrödinger equation in units $2m = \hbar = 1$,

$$\frac{d^2\psi}{dx^2} + [E - V(x)]\,\psi = 0. \tag{6}$$

However, the Regge-Wheeler and Schrödinger equations differ in their boundary conditions.

The Regge-Wheeler potential is a barrier. It vanishes at the horizon $r = 2M$ because of the factor $1 - \frac{2M}{r}$, and it vanishes at spatial infinity because it decreases according to $\frac{\ell(\ell+1)}{r^2}$ as $r \to +\infty$. Between these limits, the Regge-Wheeler potential has a maximum close to $r = 3M$, which is a region called the photon sphere. This is the radius where massless particles can orbit the black hole on unstable circular null paths. In the large ℓ limit, the peak of the potential approaches this radius exactly. The resulting problem is therefore a scattering problem on the line $-\infty < r_* < \infty$.

Zerilli later showed that even-parity perturbations can also be written in a Schrödinger-type form, but with a different effective potential [2]. Chandrasekhar and Detweiler found that the odd- and even-parity Schwarzschild perturbation spectra are the same [3]. Then, for the quantum mechanical discussion of Schwarzschild perturbations, it is sufficient to study just the Regge-Wheeler potential.

QNM BOUNDARY CONDITIONS

For a standard quantum bound state, the wavefunction must be square-integrable:

$$\psi(x) \to 0 \quad \text{as} \quad x \to \pm\infty. \tag{7}$$

This condition ensures that the Hamiltonian is Hermitian under the usual inner product and gives real, discrete energies.

For an ordinary scattering problem with real $E = k^2$, we can specify an incoming wave and solve for the reflected and transmitted waves. For a barrier that vanishes at both ends, a wave incident from the left has the asymptotic form

$$\psi(x) \sim e^{-ikx} + R(k)e^{ikx}, \qquad x \to +\infty, \tag{8}$$

$$\psi(x) \sim T(k)e^{-ikx}, \qquad x \to -\infty, \tag{9}$$

where R and T are reflection and transmission amplitudes. For real k, this is a unitary scattering problem.

The condition for QNMs is different. There is no incident wave from infinity and no wave coming out of the past horizon. For black holes, the mode must be outgoing at spatial infinity and ingoing at the horizon. With $\Psi = e^{i\omega t}\psi(r_*)$, this means

$$\psi(r_*) \sim e^{-i\omega r_*}, \qquad r_* \to +\infty, \tag{10}$$

$$\psi(r_*) \sim e^{+i\omega r_*}, \qquad r_* \to -\infty. \tag{11}$$

The first gives $e^{i\omega(t-r_*)}$, a wave moving toward larger r_*. The second gives $e^{i\omega(t+r_*)}$, a wave moving toward smaller r_* and therefore into the horizon.

These boundary conditions give the resonance condition: there is an outgoing wave on each side of the barrier but no incoming wave. Since ω is complex, the spatial part of the mode is not normalizable. If $\omega = \omega_R + i\Gamma$ with $\Gamma > 0$, then Eq. (10) grows as $e^{\Gamma r_*}$ at spatial infinity and Eq. (11) grows as $e^{-\Gamma r_*}$ near the horizon. This divergence is the behavior of resonance eigenfunctions after imposing purely outgoing boundary conditions.

The main takeaway is that Eq. (4) may look Hermitian, but the QNM boundary conditions make the eigenvalue problem non-Hermitian. Thus, the spectrum of quasinormal modes is discrete and complex, rather than real.

WKB APPROXIMATION

The semiclassical approximation gives a direct way to estimate the QNM frequencies. For

$$\frac{d^2\psi}{dx^2} + Q(x)\psi = 0, \qquad Q(x) = \omega^2 - V(x), \tag{12}$$

the local wavenumber is

$$k(x) = \sqrt{Q(x)} = \sqrt{\omega^2 - V(x)}. \tag{13}$$

Away from turning points, the leading WKB solutions are

$$\psi_{\rm WKB}(x) \simeq \frac{C_\pm}{\sqrt{k(x)}} \exp\left(\pm i \int_{x_{\rm ref}}^{x} k(x')\,dx'\right), \tag{14}$$

where $x_{\rm ref}$ is an arbitrary reference point [4].

For the fundamental mode and first few overtones, ω^2 is close to the maximum of the effective potential barrier. Here, the fundamental mode means the $n = 0$ mode, and the first few overtones are the modes with $n = 1, 2, \ldots$. In the Schrödinger-like equation, ω^2 is close to the maximum value of $V(x)$.

Let x_0 be the position of that maximum and define

$$V_0 = V(x_0), \qquad V_0'' = \left.\frac{d^2V}{dx^2}\right|_{x_0} < 0. \tag{15}$$

Near the maximum, we can expand:

$$V(x) = V_0 + \frac{1}{2}V_0''(x - x_0)^2 + \cdots. \tag{16}$$

In an ordinary WKB barrier problem, the classical turning points are the points where

$$\omega^2 - V(x) = 0. \tag{17}$$

If these points are well separated, we can match the WKB solution across the left turning point, propagate it through the middle region, and then match it again across the right turning point. For black hole QNMs, as previously mentioned, the lowest overtones have frequencies with ω^2 close to the barrier height. In this case, the two turning points are not well separated and nearly merge, which means the WKB connection formulas are no longer the correct approximation.

Instead, we expand the potential near its maximum and match the wavefunction across the entire barrier peak region at once. Schutz and Will carried this out to leading order, and Iyer and Will extended the same idea to higher order [5, 6].

At lowest order, the QNM quantization condition is

$$\frac{\omega^2 - V_0}{\sqrt{-2V_0''}} = i\left(n + \frac{1}{2}\right), \qquad n = 0, 1, 2, \ldots. \tag{18}$$

Thus

$$\omega_n^2 \simeq V_0 + i\left(n + \frac{1}{2}\right)\sqrt{-2V_0''}. \tag{19}$$

The barrier height sets the main oscillation scale, while the curvature controls the energy leakage rate. A slowly varying barrier gives smaller damping, while a sharply curved barrier gives larger damping.

This equation motivates a correspondence between QNM resonances and bound states of an associated potential well. In an ordinary potential well, the WKB quantization comes from matching decaying tails to oscillatory motion between turning points. For a QNM, the matching is instead done across a potential maximum and the boundary conditions are outgoing. The resulting quantization condition has a structure similar to the ordinary bound-state quantization, including the factor of $n + \frac{1}{2}$, but with the frequency continuing into the complex plane.

THE INVERTED POTENTIAL METHOD

In 1984, Ferrari and Mashhoon exactly defined the correspondence between QNMs and bound states for analytically solvable potentials [7]. Consider a barrier problem

$$\frac{d^2\psi}{dx^2} + \left[\omega^2 - V(x, P)\right]\psi = 0, \tag{20}$$

where P denotes the parameters of the potential. Suppose that the potential has the transformation property

$$V(x, P) = V(-ix, P'), \qquad P' = \mathcal{T}(P), \tag{21}$$

where $\mathcal{T}$ denotes the parameter map $P \mapsto P'$.

Under the complex coordinate transformation $x \to -ix$, the QNM factors of $e^{\pm i\omega x}$ become exponentials $e^{\pm\omega x}$. For the conditions listed in Eqs. (10) and (11), the transformation $r_* \to -ir_*$ gives

$$\psi(r_*) \sim e^{-\omega r_*}, \qquad r_* \to +\infty, \tag{22}$$

$$\psi(r_*) \sim e^{+\omega r_*}, \qquad r_* \to -\infty. \tag{23}$$

Thus, the outgoing boundary conditions of the barrier problem are mapped into decaying boundary conditions of an ordinary bound-state problem.

The differential equation itself also changes:

$$\psi'' + \left[\omega^2 - V(r_*, P)\right]\psi = 0$$

is transformed into

$$\Phi'' + \left[-\omega^2 + V(-ir_*, P)\right]\Phi = 0.$$

If $V(-ir_*, P) = V(r_*, P')$, this becomes

$$\Phi'' + \left[-\omega^2 + V(r_*, P')\right]\Phi = 0.$$

Equivalently, this is a bound-state equation in the inverted potential $-V(r_*, P')$, with energy $E = -\omega^2$.

It is common to write the negative bound-state energy as $E_n = -\Omega_n^2$, so that Ω_n is the bound-state eigenvalue parameter. The QNM frequency is then obtained from Ω_n after applying the inverse parameter transformation.

In this form, the transformed equation is the ordinary Schrödinger eigenvalue problem for bound states in the inverted potential well $-V$. It is important to understand that the black hole potential itself has not become a physical quantum well. Instead, the QNM eigenvalue problem for the potential has been mapped to a bound-state eigenvalue problem for an associated inverted potential. When the parameter transformation is carried out analytically, the QNM frequencies can be obtained analytically by continuing the bound-state spectrum back to the original barrier problem.

For the exact Regge-Wheeler potential, the problem is that the bound-state spectrum of the inverted potential is not known in closed form. Ferrari and Mashhoon therefore approximated the potential by those with bound states that are known analytically. This approximation is done by matching the solvable potential to the Regge-Wheeler potential near its maximum.

THE PÖSCHL-TELLER POTENTIAL

A useful example is the Pöschl-Teller potential, because both its bound-state spectrum and its resonance structure are known analytically [10, 13]. The Pöschl-Teller barrier is

$$V_{\mathrm{PT}}(x) = \frac{V_0}{\cosh^2[\alpha(x - x_0)]}. \tag{24}$$

Here V_0 is the height of the barrier, x_0 is the location of the maximum, and α controls the width. It is matched to the Regge-Wheeler barrier by requiring the same height and curvature at the maximum:

$$V_{\rm PT}(x_0) = V_0, \qquad \left.\frac{d^2 V_{\rm PT}}{dx^2}\right|_{x_0} = V_0''. \tag{25}$$

Since

$$\frac{d^2}{dx^2}\left.\frac{V_0}{\cosh^2[\alpha(x-x_0)]}\right|_{x=x_0} = -2\alpha^2 V_0, \tag{26}$$

the matching condition gives

$$\alpha^2 = -\frac{V_0''}{2V_0}. \tag{27}$$

Now, we invert the barrier. For simplicity, we can shift the coordinates so that the maximum is at $x_0 = 0$. Then, the bound-state equation for the Pöschl-Teller well is

$$\frac{d^2\phi}{dx^2} + \left[\frac{V_0}{\cosh^2(\alpha x)} - \Omega^2\right]\phi = 0, \tag{28}$$

where $E = -\Omega^2 < 0$. The boundary condition is

$$\phi(x) \to 0 \qquad x \to \pm\infty. \tag{29}$$

With the change of variables

$$z = \tanh[\alpha(x - x_0)],$$

the bound-state equation becomes

$$(1-z^2)\frac{d^2\Phi}{dz^2} - 2z\frac{d\Phi}{dz} + \left[\frac{V_0}{\alpha^2} - \frac{\Omega^2/\alpha^2}{1-z^2}\right]\Phi = 0.$$

This has the standard form of the associated Legendre equation,

$$(1-z^2)y'' - 2zy' + \left[\nu(\nu+1) - \frac{\mu^2}{1-z^2}\right]y = 0,$$

provided

$$\frac{V_0}{\alpha^2} = \nu(\nu+1), \qquad \mu = \frac{\Omega}{\alpha}.$$

Normalizability as $x \to \pm\infty$, equivalently as $z \to \pm 1$, imposes the condition

$$\mu = \nu - n, \qquad n = 0, 1, 2, \ldots, \quad n < \nu. \tag{30}$$

Thus the bound-state spectrum of the Pöschl-Teller well is

$$\Omega_n = \alpha(\nu - n), \qquad \nu = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{V_0}{\alpha^2}}. \tag{31}$$

The QNM spectrum follows by analytic continuation from the well back to the barrier. With the $e^{i\omega t}$ convention defined earlier, the result is

$$\omega_n = \pm\sqrt{V_0 - \frac{\alpha^2}{4}} + i\alpha\left(n + \frac{1}{2}\right), \qquad n = 0, 1, 2, \ldots \tag{32}$$

Equation (32) is exact for the Pöschl-Teller barrier and approximate for the Regge-Wheeler potential after matching V_0 and V_0''.

For the gravitational $\ell = 2$ Regge-Wheeler potential with $M = 1$, the maximum occurs at

$$r_0 = \frac{9 + \sqrt{17}}{4}, \tag{33}$$

and the matched parameters are approximately

$$V_0 \simeq 0.1513, \qquad \alpha \simeq 0.1811. \tag{34}$$

Eq. (32) gives

$$\omega_0 \simeq 0.378 + 0.0905i. \tag{35}$$

For comparison, direct numerical calculations of the Schwarzschild QNM eigenvalue problem give

$$\omega_0 \simeq 0.3737 + 0.0890i$$

which agrees well with our approximation [10]. This approximation worsens for higher overtones because they are affected by more of the potential away from the peak, and matching to only the peak is no longer sufficient.

THE ECKART POTENTIAL

The Eckart potential can be used to approximate the Regge-Wheeler potential more accurately, since it has one more shape parameter than the Pöschl-Teller potential [8]. The barrier can be written as

$$\begin{aligned} V_{\rm E}(x) = V_0 e^{2\mu} \\ - V_0\cosh^2\mu\,\{\tanh[\alpha(x-x_0)+\mu] - \tanh\mu\}^2. \end{aligned} \tag{36}$$

Here α controls the width of the barrier, x_0 sets the location of the peak, and μ is an asymmetry parameter. It has the limiting values

$$V_{\rm E}(-\infty) = 0, \qquad V_{\rm E}(+\infty) = 2V_0\sinh(2\mu), \tag{37}$$

and its value at $x = x_0$ is $V_0 e^{2\mu}$.

In the special case $\mu = 0$, the expression reduces to the symmetric Pöschl-Teller barrier,

$$V_{\rm E}(x)\big|_{\mu=0} = \frac{V_0}{\cosh^2[\alpha(x-x_0)]}. \tag{38}$$

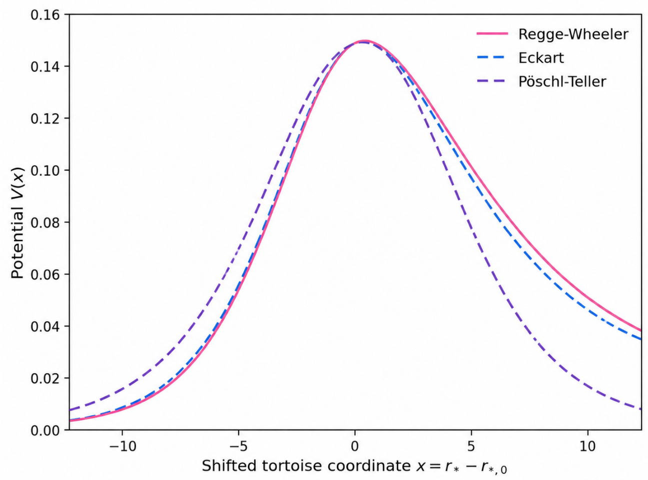

FIG. 1. The Regge-Wheeler potential plotted with the Pöschl-Teller and Eckart barrier approximations, using the shifted tortoise coordinate $x = r_* - r_{*,0}$.

The Eckart potential is useful because the extra asymmetry parameter allows the two sides of the barrier near the peak to have different shapes. This better captures the local asymmetry of the Regge-Wheeler potential, even though it still does not reproduce the exact asymptotic behavior of the full potential. The Eckart and Pöschl-Teller approximations compared with the Regge-Wheeler potential can be seen in Figure 1.

The inverted Eckart potential is also analytically solvable. The bound-state spectrum can be expressed as

$$\Omega_n = A_n + \frac{V_0 \sinh(2\mu)}{4A_n}, \tag{39}$$

where

$$A_n = \sqrt{V_0 \cosh^2 \mu + \frac{\alpha^2}{4}} - \alpha\left(n + \frac{1}{2}\right). \tag{40}$$

The allowed bound states are those for which A_n is large enough for the wavefunction to decay at both ends. Analytic continuation gives the corresponding QNMs. Defining

$$a = \sqrt{V_0 \cosh^2 \mu - \frac{\alpha^2}{4}}, \qquad b = \alpha\left(n + \frac{1}{2}\right), \tag{41}$$

we obtain

$$\omega_n = \pm a + ib + \frac{V_0 \sinh(2\mu)}{2(\pm a + ib)}. \tag{42}$$

It is useful to rewrite this result in a way that separates the oscillation frequency from the damping rate. Defining

$$\Lambda = \frac{V_0 \sinh(2\mu)}{2(a^2 + b^2)}, \tag{43}$$

the frequency can be written as

$$\omega = \omega_R + i\Gamma, \qquad \omega_R = \pm a(1 + \Lambda), \qquad \Gamma = b(1 - \Lambda). \tag{44}$$

Note that Λ is not constant, but depends on the overtone number through

$$b = \alpha\left(n + \frac{1}{2}\right).$$

As n increases, b increases and the denominator of Λ increases, so Λ becomes smaller for higher overtones. For the positive-μ case, if $\Lambda > 0$, the real part of the frequency moves closer to $\pm a$ as n increases. For positive frequencies, $\omega_R = a(1 + \Lambda)$ decreases toward a.

This feature of the Eckart approximation is missing from the Pöschl-Teller approximation. In the Pöschl-Teller limit, $\mu = 0$, so $\sinh(2\mu) = 0$ and hence $\Lambda = 0$. The Pöschl-Teller formula then gives $\omega_R = \pm a$ as independent of n, and only the damping rate changes with overtone number.

The Eckart potential tells us that as the overtone number n increases, the real part of the black hole QNM frequency decreases, while the damping rate increases. Blome and Mashhoon emphasized that this behavior agrees with numerical studies of Schwarzschild QNMs [8].

One drawback of the inverted Eckart potential is that it approaches a negative constant on one side, so it supports fewer bound states than the inverted Pöschl-Teller potential. While the Eckart approximation gives a better description of low-overtone QNMs, it still does not solve the full Regge-Wheeler problem, which can instead be approached numerically.

VÖLKEL'S NUMERICAL EXTENSION

One important limitation of the inverted potential method is that the bound-state spectrum of the inverted potential must be known in closed form. This is why earlier applications of the method replace the Regge-Wheeler potential by a solvable approximation, such as the Pöschl-Teller or Eckart potential. Völkel's 2022 paper extends the inverted potential method by showing how the bound-state spectrum can instead be computed numerically and then analytically mapped back to the QNM problem [11].

Völkel's method keeps the same basic idea as the inverted potential method, but removes the need for an exact bound-state formula. The numerically computed bound states used here should not be interpreted as physical states of the black hole. They are ordinary bound states of a constructed potential well, obtained by inverting the barrier that appears in the Regge-Wheeler equation.

The inverted potential method needs an analytic expression for the bound-state energies before one can relate them to QNM frequencies. Völkel's solution is to compute the bound-state energies numerically for several nearby choices of the potential parameters, then fit these results by a local analytic formula. That fitted formula can then be evaluated after applying the same complex parameter transformation used in the original inverted potential method.

For the Regge-Wheeler potential, Völkel introduces a scaling parameter λ multiplying the potential. This parameter acts as a bookkeeping device, where changing its sign tracks the replacement of the barrier by the corresponding well. After the numerical bound-state spectrum has been fitted, he undoes this parameter substitution and evaluates the result at the physical Schwarzschild black hole value of $\lambda = 1$. In this way, the calculation starts from a numerically stable bound-state problem but produces an approximation to the physical QNM frequencies.

Völkel first tested the procedure on the Pöschl-Teller potential, where the exact answer is already known, and verified that the method reproduces the expected QNM frequencies. He then applied the same procedure to other approximate potentials matched to the Regge-Wheeler potential. These alternatives improved on the simple Pöschl-Teller approximation, especially for the first overtone.

Völkel's 2025 paper goes further by studying the numerically calculated bound states of the inverted Regge-Wheeler potential itself [12]. It is important to understand that these states are not the physical QNM wavefunctions of the black hole. They are auxiliary bound states associated with the inverted potential correspondence. However, their spectrum and localization properties still give useful information about QNM overtones. Völkel found that as the overtone number n increases, the corresponding bound states become increasingly weakly bound. Their energies approach zero approximately exponentially with n, rather than following a hydrogen-like scaling of $E_n \sim -1/n^2$.

This result is useful for interpreting overtones. The fundamental mode is largely controlled by the region near the peak of the Regge-Wheeler potential, which lies close to the photon sphere at $r = 3M$. Higher overtones are less local: in the inverted potential picture, large-n bound-state wavefunctions spread over a much wider range of x. They are therefore sensitive not only to the height and curvature of the potential near its maximum, but also to the behavior of the potential further away from the peak. This sensitivity is relevant for non-isolated black holes: surrounding matter, such as an accretion disk, can perturb the effective potential and shift the QNM spectrum [14, 15].

APPLICATIONS

The main astrophysical application of black hole QNMs is in analyzing the ringdown stage of a black hole merger. After two black holes merge, the final remnant black hole settles, or "rings down", by emitting gravitational waves over a spectrum of primarily low-overtone QNMs. The frequencies and damping times of these modes are fixed by the parameters of the final black hole, so measuring ringdown can be used to infer the remnant's mass and spin. This is the basic idea of black hole spectroscopy [9, 10].

QNMs also provide a way to test the no-hair theorem in general relativity, which states that an isolated, stationary black hole is described completely by its mass, angular momentum, and charge. Therefore, if multiple QNM frequencies and damping times are measured from the same black hole, they should all correspond to the same mass, angular momentum, and charge. A mismatch could point to environmental effects, measurement limitations, or possibly new gravitational physics [9].

CONCLUSION

Black hole QNMs are not ordinary bound states due to their boundary conditions: waves can escape to infinity and fall into the black hole horizon. However, they can be put into the form of a one-dimensional Schrödinger equation, and tools from quantum mechanics can give useful insight into the QNM spectrum. The WKB approximation relates the lowest modes to the height and curvature of the potential barrier, and the inverted potential method relates barrier resonances to auxiliary bound states. Black hole QNMs are not physically bound states, but their interesting structure can be studied using the quantum mechanical approach to bound states.

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